A Pathway to Equitable Math Instruction

Connecting Critical Intersections

The interconnectedness of English language learning and the development of mathematical thinking

STRIDE

4

Connecting Critical Intersections

This tool was created to address the need for English learner (EL) supports in enhancing English proficiency, including language scaffolding, student talk, and clarifying their understanding of mathematical concepts. This tool seeks to assist teachers with classroom actionables in the form of scaffolding and supports that model equitable instructional practices for ELs, and foster rigor to develop language and mathematical conceptual understanding. It specifically addresses considerations around educator biases and beliefs and the diversity that exist among ELs. These issues are key to their success in both developing academic English proficiency and deepening their understanding of the mathematical concepts that were not addressed adequately in prior learning experiences.

THEMES

Teacher Beliefs

Content + Conceptual Understanding

English Language Development + Scaffolding

Teacher Support

GUIDING PRINCIPLES

Equitable access to grade-level priority math standards.

Learning opportunities for students to engage with the standards for mathematical practice.

Targeted curricula and practices designed to create equitable access to math instruction for students gaining English proficiency.

Assets-based formative assessments to inform instruction.

HOW TO USE THIS TOOL

This tool encourages teachers to **nurture opportunities for English learners to share their thinking and clarify their understanding of mathematical concepts**. It highlights effective instructional approaches in content areas, and identifies language scaffolding practices that support ELs. The grade-level samples exemplify how teachers can enhance instruction around the identified content priority standards. The general template that follows can be used to recreate a similar tool with an additional content standard. While this tool does not address all aspects of supporting ELs, it does seek to bridge the alignment of best practices, California English Language Development (ELD) standards, and mathematical priority content.

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Introduction

Cultural and Linguistic Assets

Students gaining proficiency in English, also known as English learners or ELs, have immeasurable assets stemming from their multicultural and multilingual experiences. These assets include, but are not limited to, enhanced communication skills, better understanding of how language works, greater executive function, considerable cognitive flexibility, improved memory, more opportunity to develop multiple perspectives, better sense of self, and substantial empathy and tolerance for others. As students grow up they can also have increased career opportunities and less mental decline in old age (Pavlenko 2014).

These assets support the potential to attain high levels of achievement, in both English proficiency and academically. This is most commonly evidenced by English learners who entered the school system with limited English proficiency, but were "Reclassified Fluent English Proficient" (RFEP). These former English learner students are among the best performing students in California (Hill 2014); in many instances, closely matching or outperforming their English-only (EO) peers. In 2018-19, CAASPP reports that 56.45% of English-only students who took the Smarter Balanced Summative Assessments for English language arts/literacy scored in the "Met" or "Exceeded Standard" range. That group was outperformed by their RFEP peers, of which 60.07% scored in the same range. In mathematics, 44.37% of English-only students tested scored in the "Met" or "Exceeded Standard" range, compared to 43.10% of RFEP students.

However, for those students who remained in the English learner program, the data was far below both those of EO and RFEP students. In ELA/literacy, 12.81% of EL students tested scored in the Met or Exceeded Standard range, and 12.58% met the same range in mathematics. Further examination of teaching practices reveals the lack of educators' understanding of English learners' linguistic needs—resulting in the underachievement of ELs in content areas, specifically in mathematics. The widened achievement gaps are evidenced by how instructional practices focus on teaching vocabulary, but not in conjunction with many other higher order thinking skills that enable English learners to "actively use mathematical language to communicate about and negotiate meaning for mathematical situations" (Moschkovich 2010). This tool will support teachers in providing instruction that addresses both language and mathematical development.

Effects of Systemic Racism

The patterns of underachievement historically seen among students gaining proficiency in English, many of which are also students who identify as Black, Indigenous, and people of color (BIPOC), are effects of a system that fails to see the languages and cultures our students bring to their education as assets for their own learning. Their language and culture are neither seen as an important contribution to their learning communities, nor to the rich diversity of America. The systems that uphold these white supremacist practices are most clearly seen in policies that devalue students' home languages and cultures, and instead force assimilation through English-only initiatives and the abundant shower of white-only literature and learning materials. Further evidence is seen in the access English learners have to higher-level courses in junior high and high school, especially in the area of mathematics. This supported the decision to focus this tool on mathematics equity for English learners in grades 6–8.

Tool for Supporting Priority Math Content for Students Gaining Proficiency in English in Grades 6–8

The TODOS position paper, The Mo(ve)ment to Prioritize Antiracist Mathematics: Planning for This and Every School Year, explains:

> "An antiracist position in mathematics education is a pledge to dismantle systems and structures that maintain racism within teaching and learning mathematics from challenging belief systems that perpetuate microaggressions to disrupting the role mathematics classes play in pushing students out of schooling. If we truly believe that we are moving towards assets-based views of students, we must expand our understanding of what it means to be good at mathematics, make space for alternative ways of knowing and doing mathematics based in the community, and acknowledge the brilliance, both in mathematics and beyond, of BIPOC in our classrooms. We must be explicitly antiracist."

This document was created to address the lack of language scaffolding used to nurture opportunities for students gaining proficiency in English, share their thinking, and clarify their understanding of mathematical concepts. The tool in this document seeks to support teachers with classroom actionables that model equitably instructional practices for ELs. This document specifically addresses considerations around educator biases and beliefs, as well as the variabilities that exist among English learners, as these issues are key to EL's success in developing academic English proficiency, while gaining the mathematical concepts that were not gained by past practices. Our tool will support teachers with examples of actionables that support rigor, as well as scaffolds and supports for simultaneous development of language and math content in order to support equitable practices for ELs in mathematics.

Teacher Biases and Beliefs

The impact that teachers' beliefs have on equitable practices can not be understated. In her paper titled, Teacher's Knowledge and Beliefs About English Learners and Their Impact on STEM Learning, Julie Bianchini explains that teachers' "efforts to construct safe classroom communities and effectively implement reform-based strategies have been found to impact both ELs' views of themselves as learners and their mathematics and science achievement." Her review of research on the topic of teacher beliefs highlighted the mis/connections across teachers' knowledge and beliefs, their instructional practices, and the implications for EL student learning.

One such mis/connection was the deficit-based views teachers had in regards to the EL students they service. One example of this view is seen when teachers do not implement practices that develop students' language acquisition, and teachers confuse emerging English language proficiency with the absence of STEM content understanding. Teachers often assume ELs as lacking relevant prior knowledge, experiences, and/or language, when in fact in the early stages of second language acquisition, learners spend time in the "quiet phase" as they take in and process their understanding of their second language. One deficit-based view shared by teachers was seeing ELs as being unable or unwilling to communicate with teachers and/or with their non-EL peers.

Teacher biases included stereotypes of ELs grounded in their first language, ethnicity, and/or country of origin. These stereotypes led some teachers to have low or unreasonably high expectations for their EL students. Bianchini goes on to describe one scenario in which "researchers found that teachers valued ELs' hard work and motivation over their mastery of content, which was often neither facilitated nor checked. Teachers' knowledge and beliefs resulted in ELs' placement in lower-track classes, limited support for ELs in classroom interactions, and low expectations for ELs' learning of content. As such, ELs were denied access to academic supports and adequate college preparation, even though this denial was cloaked in teachers' well-meaning concern."

The tool created for this document seeks to challenge these beliefs as it contains actions that support students gaining proficiency in English through error analysis of common misconceptions. Students study a sample problem centered on a priority math standard. Through analysis of an example and non-example solution, students will explain, conjecture, and justify their reasoning. Students are supported through rigorous activities and encouraged to build agency through productive struggle.

One-Size-Fits-All Approach Does Not Work

English learner variability is the rule, not the exception. There is no such thing as a "typical" English learner student. We use the term "English learner" as the briefest way to convey meaning and to describe a student who is adding English to their existing linguistic assets, but it does not adequately capture the range of EL types in your classroom. Each EL student is unique in their place on the language acquisition continuum and in their place in the world. Some are highly schooled in their primary language, others have experienced interrupted schooling, and many may be recent immigrants. EL Students may also be dually identified as a student with disabilities or gifted and talented—in some cases, both. Gifted EL students are frequently overlooked in the classroom because of the limitations of verbal assessments and language access barriers. In addition to the EL student types provided in the table below, EL students bring diverse life experiences. Some of these may include internationally adopted EL students, unaccompanied EL students, EL Students from mixed-status families, EL students experiencing homelessness, EL students in foster care, and LGBTQ EL students. They are all accompanied by the gift of another language, experience, and culture, along with the potential to broaden the horizons of every class they attend. Further, increased executive function and the ability to switch focus of attention, reason about other points of view, and reflect on the structure of language are all learning assets that EL students bring to the classroom (*Promoting the Educational Success*... 2017). Metacognitive (learner awareness about learning) and metalinguistic (language learner awareness about language) skills can greatly enhance the learning experience and provide mathematics teachers with a head start in nurturing critical thinking skills needed for mathematical reasoning.

This tool is designed to assist mathematics teachers in designing lessons that support all EL students and their mathematical experiences. Below is a table of different types of EL students educators may have in their classroom, also known as EL Typologies. This table is by no means exhaustive; but it can provide some insights, not just into a student's educational experience or needed level of linguistic support, but a way to consider more deeply their life experience and worldview. When teachers invest time in expanding their own knowledge about the language and culture of a student, and observe how that same student's language and culture have formed their unique identity, a much clearer instructional support path emerges. (Paris and Alim 2017).

EL TYPOLOGIES

	NEWCOMER (WELL-EDUCATED)	NEWCOMER (UNDERSCHOOLED)	LONG-TERM ENGLISH LEARNER (LTEL)
Time in US Schools	In US schools ≤ 3 years	In US schools ≤ 6 years	In US schools ≥ 6 years often born in the US
Previous Schooling Experience	Schooling from home country—usually strong	Limited or no schooling in home country	Limited or no schooling in home country—often in US schools since kindergarten
Background Knowledge of US Culture	Limited knowledge of US culture	Limited knowledge of US culture	Moderate to strong knowledge of US culture
Background Knowledge of School Culture	Good knowledge of school culture, though may be different from US schools	Limited knowledge of school culture	Strong knowledge of school culture
L1 (Primary Language) Skills	Strong oral and literacy skills in L1	Strong oral skills in L1, Limited or no literacy skills in L1	Most have moderate to strong oral skills in L1, Some have literacy skills in L1, many do not
Oral English Skills	Limited oral skills	Limited oral skills and slow acquisition of English	Usually fluent orally in English
Literacy Skills (Reading & Writing)	Limited literacy skills in English, but rapid acquisition of language, Usually highly motivated	Limited literacy skills in English, struggles with academics	Limited literacy skills in English, struggles with academics, Often mismatch between student's own perception of skills and actual skill level

Tool for Teacher Use

The table with sample actionables below is labeled as the tool portion of this document. The tool was created for teachers to use to address the need for English learners to engage in rigorous language and mathematical learning in order for them to progress towards proficiency. Additionally the tool includes examples of scaffolding and supports for simultaneous language and content development. The scaffolding activities were selected to increase student discourse. The areas that follow detail the considerations made when selecting elements to include in the tool.

Need for Rigor

In contrast to the low expectations that some teachers have in regards to the abilities of ELs, both the "Key Instructional Principles and Practices" found in the 2016 "Framework for Re-envisioning Mathematics Instruction for English Learners" and the "Areas of Focus" outlined in English Learner Success Forum's "Guidelines for Improving Math Materials for English Learners" call for ELs to engage in rigorous grade-level tasks and assignments in which they are growing their conceptual understanding through productive struggle. The struggle is productive because students are supported using strategic scaffolds designed to grow their conceptual understanding.

Teachers often assume ELs lack relevant prior knowledge, experiences, and/or language, so they cut off access to grade-level content until those "gaps are filled." Instead, research supports that by leveraging students' funds of knowledge they build understanding around rigorous content. For example, there are increased benefits to students when teachers encourage and support students' use of home language and translanguaging to support mathematical reasoning.

Making connections to students' home life and relevant mathematics is incredibly important as students adjust to remote learning. In their publication, "Do's & Dont's of ELL Instruction", the EL Success Forum calls on teachers to design lessons that prompt students to make connections to their homes and families.They stress the importance of encouraging students to read, write, listen, and speak about mathematics with family members and siblings.

Another important consideration when focusing student work around rigor is the use of content and language grade-level standards. When students are seen as not capable there may be a tendency to lower expectations and keep students working with below grade-level standards. The pandemic and the distance-learning environment imposed upon schools has further highlighted this issue of rigor. The Council of the Great City Schools' "Addressing Unfinished Learning after COVID-19 School Closures" document urges educational leaders to ensure that their plans to address unfinished learning do not focus on students' deficits or compromise rigor in the name of "filling gaps," thus deemphasizing college- and career-ready mathematics. Instead, the focus should be on appropriate scaffolds and supports for English learners as they develop language and content simultaneously. The "Instructional Priority Content" document by Student Achievement Partners was used to identify the priority standards that were used in the tool. The sample actions support the teaching of the priority standards.

Additionally the rigorous California English Language Development (ELD) Standards and Standards for Mathematical Practices were also included in the tool to support math teachers' understanding of the interconnectedness of mathematics concepts and language. The WestEd document, "*Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning*" was used as a resource to identify ELD standards and mathematical practices that applied to the selected priority mathematical grade level standard. The WestEd document also provided scaffolding suggestions that were enhanced to support students in developing facility and proficiency with English. As students enhance their capacity to use language for academic purposes—explain, justify, critique, argue from evidence, etc.—they are using language, not just as a communication tool, but as a thinking tool.

Create Opportunities for ELs to Speak

Oral language development is the foundation of literacy, according to the National Literacy Panel on Language-Minority Children and Youth (2006). English learners need frequent opportunities to engage in well-scaffolded, challenging, and complex discussions of rigorous academic topics in order to develop not only their oral language skills, but also other higher-order thinking skills. Unfortunately, this is not common practice in classrooms across the nation. In 2002, Diane August, a senior research scientist at the Center for Applied Linguistics, disclosed that English learners spend less than two percent of their school day in oral language development (Soto-Hinman 2011), while Pauline Gibbons (2002) divulged that when English learners speak, the majority of their speech is very shallow, mostly single word responses. Furthermore, as educators enacted the EL Shadowing protocol, developed by Ivannia Soto, to follow and observe an English learner in an academic setting at five-minute intervals for two hours, the first thing they discovered is English learners hide quietly in their class/classes all day, and they are rarely provided opportunities to speak.

According to Bunch (2013), teachers must "purposefully enact opportunities for the development of language and literacy in and through teaching…core curricular content, understandings, and activities" if they are to interest, engage, and challenge their EL students. In addition, the California English Learner Roadmap's "Policy Principle #2" emphasizes meaningful access to Common Core State Standards, a relevant and rigorous curriculum, and opportunity to develop language for English learners. As students regularly engage in examining and exploring mathematical concepts and arguments, they are **applying their linguistic knowledge and skills** to make sense of the content they are learning, as well as **incorporating mathematical practices** that enable them to not only solve the problems or seek answers to questions, but also understand the mathematical concepts deeply so they can apply them to other problems. Educators can strategically support students in learning and enhancing their language skills and knowledge via purposeful activities, which include language development routines that cultivate discussions in pairs and small groups, maximizing each student's opportunity to use language for academic purposes. In mathematics, the use of whole-group discussions for the purpose of comparing, contrasting, and connecting the variety of ways of thinking about the mathematical concepts in the lessons enable the students to not only develop their academic content, but also the mathematical discourse and their English proficiency. And, recently, the EL Success Forum remote-learning recommendations include the use of technology (word processing, audio recording, drawing, speech-to-text, or screencasting tools) strategically, to comment on students' assignments and provide feedback on how to improve their writing, the most complex form of language.

This document responds to the need to provide English learners with opportunities to develop academic discourse, not only in mathematics, but also in other subject areas. Through our actionable examples and scaffolds, English learners can be actual and active participants in their classroom's complex discussions and discoveries of academic content.

Tool Application

This tool helps teachers nurture opportunities for EL students to share their thinking and clarify their understanding of mathematical concepts. It highlights key content areas and identifies language scaffolding practices that support EL students. The grade-level samples show how teachers can support work around the identified content priority standard and the General Template that follows can be used to recreate the tool with an additional content standard. While this tool does not address all aspects of supporting English learners, it does seek to bridge the alignment of best practices, ELD standards, and math priority content by providing examples of activities and scaffolds that can be used to support students.

This chart details the elements included in the grade-level example tables on page 11. There is a General Template for teachers to create their own table, recreating the process for another standard, on page 26 of this toolkit.

TABLE ELEMENTS

ELEMENT	WHAT IT IS	HOW TO USE IT	WHAT IT IS NOT
Priority Content for X Grade	References priority mathematics content cluster, an Identified Priority Standard, and Standards for Mathematical Practices	The scaffolding activities that follow support instruction towards mastery of the identified math priority standard. Priority standards were selected for samples to intentionally build coherence across concepts for grades 6-8.	The tool only provides examples for one priority standard under the priority cluster. The General Template can be used to address additional standards. For considerations for addressing remaining grade-level content, refer to Instructional Priority Content for 2020-21.
Intersection of English Language Development (ELD) and Mathematical Comprehension	Identified aligned priority ELD standards: Integrating the CA ELD Standards into K-12 Mathematics and Science Teaching and Learning	Unpack standard to draw out language objectives detailed in priority ELD standards.Consider how the standard applies to mathematics teaching and learning in classrooms with EL students.	It does not address an exhaustive list of ELD standards. Differentiation across proficiency level expectations are not detailed.
Examples of OK Student Thinking and Misconceptions	This column includes the sections used to take students through error analysis of common misconceptions.	Example of how teachers can shift the focus from identifying the correct answer to conceptual understanding of the identified priority standard.	It is not an exhaustive list. Teachers can use different sample problems and non-examples.
Actionables: Why & How Scaffolding and Supports for Simultaneous Development	Actionable practices to support equitable access to math content for English learners.	Steps that can be used to support students simultaneous development of language and math content, centered on priority standards.	It is not an exhaustive list. Teachers can use different scaffolds and/or enhance those included. Differentiated by proficiency level.
Overarching Instructional Guidelines	Guidelines that aided in the selection of the actionables. The scaffolds and supports selected align with overarching guidelines to support students understanding around priority content. Guidelines are they why behind the selection of actions.	These can be referenced to understand the purposes for actionables. The sources can also be further explored by teachers for more context.	It is not an exhaustive list. Teachers can use other guidelines to focus their instruction to meet the needs of their students.
Sample Starter Problem	This row contains a problem that addresses the identified priority standard. There can be several varying sample problems.	Students will analyze the answers to this problem. Students will work with a correct solution and a misconception. Teachers are provided actionables and guidelines to support the introduction of the sample problem.	Teachers can use different sample problems, and examples and/or non-examples.
Example of Correct Student Thinking	An example of one correct solution that could address the sample starter problem. Providing students with the correct answer allows their focus to shift to understanding the concepts and communicating their understanding.	Teachers can use this as a correct example of a mathematical solution for the priority standard to support error analysis of the misconception. Teachers should model multiple methods for students and elicit those from students.	Teachers can use different sample problems, and correct student thinking.
Non-example (Misconception) of Student Thinking	A common misconception students have in their attempts to solve the sample starter problem.	Teachers can use the non-example/ misconception to focus error analysis.	Teachers can use different non-examples/misconceptions.
Additional Scaffolding and Supports	Other scaffolds that can be used outside of analysis of Sample Starter Problem	These can be used to support priority standard understanding that is not connected to the error analysis process.	It is not an exhaustive list of scaffolds.

Grade 6 Sample

The information that follows provides an example of how the identified priority math standards that fall under the identified priority clusters can be addressed to support students gaining proficiency in English.

GRADE 6 MATH STANDARD 6.RP.A.1

Priority Content for 6th Grade

Content Cluster: 6.RP.A

Math Standard Addressed: 6.RP.A.1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

Examples: "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." or "For every vote candidate A received, candidate C received nearly three votes."

Standards for Mathematical Practice

SMP 1 - Make sense of problems and persevere in solving them.

SMP 2 - Reason abstractly and quantitatively.

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP 6 - Attend to precision.

Intersection of English Language Development (ELD) and Mathematical Comprehension

Align ELD and Standards for Mathematical Practice

Standards for Mathematical Practice
Working collaboratively provides students the opportunity to use language while examining the problem presented (SMP 1), offering their opinions, and asking and answering questions (SMP 2) to develop their understanding of the mathematical concepts deeply as they solve cases (SMP 6). As students present their solutions, they adapt their language choices, as well as use specific nouns and noun phrases, adding details to accurately (SMP 6) and convincingly present their arguments (SMP 3). While listening attentively to their peers, they analyze and evaluate how well their peers present and support their arguments (SMP 3).

ELD Standards

PI.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics-

PI.3 - Offering and justifying opinions, negotiating with and persuading others in communicative exchanges.

PI.4 - Adapting language choices to various contexts (based on task, purpose, audience, and text type).

IP.5 - Listening actively to spoken English in a range of social and academic contexts.

PI.6 - Reading closely literary and informational texts and viewing multimedia to determine how meaning is conveyed explicitly and implicitly through language.

PI.7 - Evaluating how well writers and speakers use language to present or support ideas.

PI.8 - Analyzing how writers and speakers use vocabulary and other language resources for specific purposes (to explain, persuade, entertain, etc.) depending on modality, text type, purpose, audience, topic, and content area.

P1.9 - Expressing information and ideas in formal oral presentations on academic topics.

PII.3 - Using verbs and verb phrases.

PII.4 - Using nouns and noun phrases.

PII.5 - Modifying to add details.

GRADE 6 MATH STANDARD 6.RP.A.1 (continued)

Examples of Correct Student Thinking and Misconceptions

Sample Starter Problem
(Math Pathways & Pitfalls: Unit 7, Lesson 1)

"Erin made pineapple punch for her math club meeting. Her recipe called for a pineapple juice to soda ratio of 2:8. She used 5 cups of pineapple juice and 20 cups of soda. Is this ratio equal to the ratio in the original recipe?"

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Introduce activities that deepen and extend learning through various modes of communication (i.e., speaking, listening, reading, writing).
2. Support productive struggle in learning mathematics. For example, give students time to struggle with tasks, and ask questions that scaffold students' thinking, without stepping in to do the work for them.
   - To develop SMP 1, ask "How could you start this problem?"
   - To develop SMP 2, ask "What does _____ mean to you? (symbol, quantity, diagram, etc.)?"
   - To develop SMP 3, ask "What math language will help you prove your answer?"
   - To develop SMP 6, ask "Explain what you did to solve the problem."
3. Help distinguish between common, everyday meanings of language and mathematical meanings, as they emerge in the lessons. Deconstruct text into meaningful chunks (pineapple punch; pineapple juice; called for; a pineapple juice to soda ratio of 2:8; etc.)
4. Clarify what 2:8 represents.
5. Invite students to compare and contrast sets of objects around their home and describe them using ratios and using the language of ratios. Examine and explore relationships (ratios) between different objects (1 chair, 4 legs), animals (1 dog, 1 tail), humans (1 person, 2 eyes).
6. After allowing time for every student to work on the sample problem, have students talk in pairs and then small groups, about the diagram in the Starter Problem and each other's work. Encourage them to use phrases such as "for every," "for each," and "compared to." Since these terms describe relationships rather than something tangible, they are best learned in context.

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

Introduce activities

- Focus I: Interdependence of Mathematical Content, Practices and Language
- Guideline 3: Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms.
- Specification 3A: Activities deepen and extend learning through various modes of communication (i.e., speaking, listening, reading, writing).

Mathematical errors

- Focus II: Scaffolding and Supports for Simultaneous Development
- Guideline 6: Guidance for anticipating potential language demands and opportunities in student activities.
- Specification 6C: Materials demonstrate activities and ways to help distinguish between common, everyday meanings of language and mathematical meanings (table, round, product, origin, similar, etc.), as they emerge in the materials.

Principles to Actions: Ensuring Mathematical Success for All (NCTM)

- Mathematics Teaching Practices: Support productive struggle in learning mathematics.

Leading with Learning (WestEd)

- Guiding Questions to develop mathematical Practices

GRADE 6 MATH STANDARD 6.RP.A.1 (continued)

Examples of Correct Student Thinking and Misconceptions

Example of Correct Student Thinking

In both recipes, the amount of juice (J) compared to the amount of soda (S) is the same, so the ratios are equal. Both are 1 cup of juice for every 4 cups of soda. She just used 2 ½ recipes. I drew a diagram to prove it.

1 recipe 2:8
JSSSS JSSSS

2 recipes 4:16
JSSSS JSSSS JSSSS JSSSS

2 ½ recipes 5:20
JSSSS JSSSS JSSSS JSSSS JSSSS

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Nurture opportunities for teacher-student and student-student interactions that model and reflect the intent of the Mathematical Practices. Model increasing the recipe based on the 2:8 ratio.
2. Include prompts for students to reflect on their thought processes, language use, methods, and learning of mathematical content. For example, use Bounce Cards:

"Bounce Cards for Intermediate Grades"

These conversation starters stress the importance of active listening and student discourse, and can be used in any content area. The cards provide three ways for students to participate in a conversation by adding on to what someone said (Bounce), summarizing or paraphrasing to add clarity (Sum It Up), or asking questions (Inquire).

Instructions:

1. Cut apart the conversation cards so one card can be given to each student in a group of three. Each person will have a role based on the card they get (Bounce, Sum It Up, or Inquire).
2. Select two students to model a conversation with you for the class, but practice with them ahead of time.
3. Model the conversation.
4. Discuss three ways to respond in conversation.
5. Allow students to practice.
6. Once students gain confidence, begin removing sentence stems and just give students cards with the titles as they begin to appropriate the stems.
7. Provide students with a prompt that requires a back and forth discussion.

Some sentence starters that students could use: *"That reminds me of…" (Bounce), "You are saying that…" (Sum it Up), "Will you tell me more about…" (Inquire).*

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (ELSF)

Nurture opportunities

- Focus V: Assessment of Mathematical Content, Practices, and Language
- Guideline 13: Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency.
- Specification 13A: Provide examples of teacher-student and student-student interactions that model and reflect the intent of the Mathematical Practices.

Bounce Cards

- Focus I: Interdependence of Mathematical Content, Practices, and Language
- Guideline 1: Strategic opportunities to use and refine both language and mathematics over time.
- Specification 1B: Materials guide teachers to encourage students to build their own understanding of mathematics actively, through sustained activities and experiences.

GRADE 6 MATH STANDARD 6.RP.A.1 (continued)

Examples of Correct Student Thinking and Misconceptions

Non-example (Misconception) of Student Thinking

The ratios aren't equal. The recipe says there needs to be 6 more cups of soda than juice, but Erin used 15 more cups of soda than juice. That's too much soda.

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Create opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.
2. Have students justify their reasoning, communicate their reasoning to others, and respond to the arguments of others. This includes explaining the reasoning behind correct answers, as well as the misconceptions behind incorrect responses, which enhances conceptual understanding of central math ideas.
3. During lessons, have students contrast correct and incorrect ways to solve a problem. Ask them to talk explicitly about why a particular misconception occurs, how to avoid it, and how to think correctly about the mathematics in the problem.
4. To clarify any lingering confusion, ask a student to illustrate the ratio of pineapple juice and soda by drawing pictures or using realia. Bring in bottles of soda and juice and make the punch in class and emphasize key terms like_____to_____ratio, compared to, and equivalent.

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (ELSF)

Mathematical errors

- Focus III: Mathematical Rigor through Language
- Guideline 7: Explicit guidance for teachers to engage students in using mathematical practices.
- Specification 7B: Teacher materials point out opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.

A Framework for Re-envisioning Mathematics Instruction for English Language Learners (CGCS)

- Encouraging Productive Struggle

Math Pathways & Pitfalls (WestEd)

- Turn Pitfalls into Pathways for Learning

GRADE 6 MATH STANDARD 6.RP.A.1 (continued)

Additional Scaffolding and Supports for Simultaneous Development

"Fractions, Factors, and Functions, Oh My! Are My ELs Attaching Meaning to Math Words?" (ELSF)

"When you think of 'mathematics and language,' words such as Hypotenuse, Polygon, Polynomial, Function, Set, Factor, and so on might come to mind. Specialized words and special ways of using those words are two of the distinguishing characteristics of mathematical language. For mathematics teachers of English learners, one common question is when and how should I introduce the specialized words that are part of mathematical language?

There is not a 'one-size-fits' all answer to this question and the ELSF guidelines 2 do not include specific guidance on how to introduce vocabulary. Instead, the ELSF guidelines highlight that curriculum materials and guidance for teachers should include activation of prior knowledge and hands-on applications to 'help students make connections between current language, new language, and mathematical concepts.'"

Overarching Instructional Guidelines

Fractions, Factors, and Functions

- Focus I: Interdependence of Mathematical Content, Practices, and Language
- Guideline 1: Strategic opportunities to use and refine both language and mathematics over time.
- Specification 1C: Materials provide strategies to help students make connections between current language, new language, and mathematical concepts.

Grade 7 Sample

The information that follows provides an example of how the identified priority math standards that fall under the identified priority clusters can be addressed to support students gaining proficiency in English.

GRADE 7 MATH STANDARD 7.RP.A.3

Priority Content for 7th Grade

Content Cluster: 7.RP.A

Math Standard Addressed: 7.RP.A.3
Use proportional relationships to solve multistep ratio and percent problems.

Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Standards for Mathematical Practice

SMP 1 - Make sense of problems and persevere in solving them.

SMP 2 - Reason abstractly and quantitatively.

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP 4 - Model with mathematics.

SMP 6 - Attend to precision.

Intersection of English Language Development (ELD) and Mathematical Comprehension

Align ELD and Standards for Mathematical Practice

Standards for Mathematical Practice
Working collaboratively provides students the opportunity to use language while examining the problem presented (SMP 1), offering their opinions, and asking and answering questions (SMP 2) to develop their understanding of the mathematical concepts deeply as they solve cases (SMP 6). As students present their solutions, they adapt their language choices, as well as use specific nouns and noun phrases, adding details to accurately (SMP 6) and convincingly present their arguments (SMP 3). While listening attentively to their peers, they analyze and evaluate how well their peers present and support their arguments (SMP 3).

ELD Standards

PI.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics.

PI.3 - Offering and justifying opinions, negotiating with and persuading others in communicative exchanges.

PI.4 - Adapting language choices to various contexts (based on task, purpose, audience, and text type).

IP.5 - Listening actively to spoken English in a range of social and academic contexts.

PI.6 - Reading closely literary and informational texts and viewing multimedia to determine how meaning is conveyed explicitly and implicitly through language.

PI.7 - Evaluating how well writers and speakers use language to present or support ideas.

PI.8 - Analyzing how writers and speakers use vocabulary and other language resources for specific purposes (to explain, persuade, entertain, etc.) depending on modality, text type, purpose, audience, topic, and content area.

P1.9 - Expressing information and ideas in formal oral presentations on academic topics.

PII.3 - Using verbs and verb phrases.

PII.4 - Using nouns and noun phrases.

PII.5 - Modifying to add details.

GRADE 7 MATH STANDARD 7.RP.A.3 (continued)

Examples of Correct Student Thinking and Misconceptions

Sample Starter Problem
(Math Pathways & Pitfalls: Unit 7, Lesson 6)

"A pickup truck traveled 8 miles in 10 minutes and a car went 22 miles in 25 minutes. Which went at a faster rate?"

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Select materials that consistently provide access to cognitively demanding tasks. Listening, speaking, reading, and writing about mathematics must be an integral part of the instruction to support students' understanding.
2. Classwork should not be limited to memorizing facts, rules, or only carrying out calculations, but should extend to showing, describing, and discussing the underlying mathematical meaning of those procedures. Provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, tables, graphs, words, etc.).
3. Support productive struggle in learning mathematics. For example, give students time to struggle with tasks, and ask questions that scaffold students' thinking without stepping in to do the work for them.
   - To develop SMP 1, ask "How could you make this problem easier to solve?"
   - To develop SMP 2, tell students to "Create a representation of the problem."
   - To develop SMP 3, ask "How can you prove that your answer is correct?"
   - To develop SMP 4, ask "How would it help to create a diagram, graph, table?"
   - To develop SMP 6, tell students to "Explain what you did to solve the problem."

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

Math Rigor

- Focus III: Mathematical Rigor through Language
- Guideline 8: Maintain appropriate challenge and high-expectations of mathematics learning for EL students.
- Specification 8A: Materials consistently provide access to cognitively demanding tasks.

Scaffolding

- Focus II: Scaffolding and Supports for Simultaneous Development
- Guideline 4: Opportunities for students to interact with and produce a variety of methods and representations.
- Specification 4A: Learning activities provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, tables, graphs, words, etc.).

A Framework for Re-envisioning Mathematics Instruction for English Language Learners (CGCS)

- Employing Rigorous Tasks and Assignments

Principles to Actions: Ensuring Mathematical Success for All (NCTM)

- Mathematics Teaching Practices: Support productive struggle in learning mathematics.

Leading with Learning (WestEd)

- Guiding Questions to Develop Mathematical Practices

GRADE 7 MATH STANDARD 7.RP.A.3 (continued)

Examples of Correct Student Thinking and Misconceptions

Example of Correct Student Thinking

The pickup went 8 miles in 10 minutes. So 8 divided by 10 is how far it went in 1 minute. For the car, I divided 22 by 25. The car went faster, 0.88 mile per minute. Besides, the car drove for 2.5 times as many minutes, but it went more than 2.5 times as many miles.

Pickup

8 miles/10 minutes = 0.8 mile/1 minute

Car

22 miles/25 minutes = 0.88 mile/1 minute

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Cultivate and facilitate back-and-forth mathematical discussions between students that refer to and build on each other's ideas.
2. Create an environment for equitable participation and risk-taking in conversations.
3. Review the concept that a rate is a comparison of two quantities, such as the distance and time of a trip, compared by division.
4. Help students realize that phrases such as "miles per hour," "cost per can," and "words per minute," are ways to express rates.
5. Revisit the distance formula (distance = rate • time, or d = r • t) to help students understand that they need to compare the distance/time for the pickup to the distance/time for the car.
6. Choose and Defend is an activity during which students are asked to choose and defend their position on a statement (true or false), provided in writing by the teacher or another student. Example of a true statement: A square is always a rectangle. Example of a false statement: If a square is a rectangle, then a rectangle is a square. The positions to choose from are:
   - I agree (with this statement).
   - I disagree (with this statement).
   - I am not sure. I have questions.

This activity can be used either at the beginning of the lesson or during a specific part of the lesson, with the ultimate goal of revealing and discussing students' thinking and/or misconceptions, and promoting students' participation in discourse (oral and written). This activity is also intended to create an opportunity for students to reveal their mathematical thinking, including potential misconceptions, through language and also numeric, symbolic, and/or visual representations.

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (ELSF)

Math Rigor

- Focus III: Mathematical Rigor through Language
- Guideline 9: Guidance for facilitating mathematical discussions and co-construction of meaning.
- Specification 9A: Materials include prompts for teachers to cultivate and facilitate back-and-forth mathematical discussions between students that refer to and build on each other's ideas.
- Specification 9C: Materials allow for equitable participation and risk-taking in conversations.

Choose and Defend

- Focus III: Mathematical Rigor through Language
- Guideline 7: Explicit guidance for teachers to engage students in using mathematical practices.
- Specification 7B: Teacher materials point out opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.
- Specification 7C: Teacher materials provide opportunities for students to revise their own, peers', and/or fictious mathematical writing.

Math Pathways & Pitfalls (WestEd)

- Mathematical Insights & Teaching Tips
- Mathematical Discussion Supports

GRADE 7 MATH STANDARD 7.RP.A.3 (continued)

Examples of Correct Student Thinking and Misconceptions

Non-example (Misconception) of Student Thinking

I need to divide to find out how many miles per minute for the pickup and the car. 10 divided by 8 is how far the truck went in 1 minute. For the car, I divided 25 by 22.

Pickup
10 miles/8 minutes = 1.25 miles/1 minute

Car
25 miles/22 minutes = 1.14 miles/1 minute

The pickup went faster.

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Reinforce the habits of analyzing mistakes and persisting through problem solving struggles. Then use examples and non-examples to guide student learning through error analysis.
2. Review the meaning of phrases such as "faster," "more time," "less time," and "faster time." A common pitfall for students is to think that a rate is faster if it takes less time, regardless of the distance. They may mistakenly equate more time with faster, even though speed and time have an inverse relationship (one increases when the other decreases when the distance is the same).

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (ELSF)

Mathematical errors

- Focus III: Mathematical Rigor through Language
- Guideline 7: Explicit guidance for teachers to engage students in using mathematical practices.
- Specification 7B: Teacher materials point out opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.

Math Pathways & Pitfalls (WestEd)

- Turn Pitfalls into Pathways for Learning

A Framework for Re-envisioning Mathematics Instruction for English Language Learners (CGCS)

- Encouraging Productive Struggle

GRADE 7 MATH STANDARD 7.RP.A.3 (continued)

Additional Scaffolding and Supports for Simultaneous Development

Teacher Discourse Moves to Facilitate and Deepen Students' Reasoning:

Giving small groups of students an intriguing question to work through together or an interesting phenomenon to explain sets the stage for students to share ideas and for teachers to support students' developing skill in careful, critical thinking and in effective communication. Teacher facilitation of students' reasoning involves listening in, and deciding if and when to highlight some ideas by revoicing them, when to point out a contradiction in their collective thinking that the students themselves did not catch, and when to model an alternate way of expressing an idea. The work always involves helping students listen carefully to one another, do their best to express their own thinking, and thinking carefully about the idea that's developing among them.

The Teacher Discourse Moves focus on:

- Help a student clarify ("What I hear you saying is…").
- Make an idea public ("This group is finding common denominators and comparing them.").
- Emphasize an idea.
- Help students listen carefully and react ("Let's all listen to her explanation of the problem so that we can ask her a question about her thinking.").
- Help students deepen their reasoning.
- Help students apply their thinking to others' ideas.

Student Discourse Moves for Collaborative and Critical Thinking:

When we present students with intriguing challenges to think through, some will jump right in and others will hold back—some, because they've had years of being trained to spit out correct answers quickly and to keep quiet if they cannot; some, because they have not had much experience in explaining their thinking; some, because they believe that students should be silent and listen; and others, because they're uncertain how to word things so others will understand.

But to activate the collaborative thinking and discussion—the engine driving language development—we need to help students learn new ways of interacting. Students need practice in collaborating to examine issues and build new understandings together.

Student Discourse Moves focus on:

- Tell and Explain
- Clarify
- Restate or Summarize
- Compare
- Support
- Build on
- Question or Challenge

Overarching Instructional Guidelines

WIDA Focus: Strengthening Reasoning, Strengthening Language

- "Teacher Discussion Moves to Facilitate and Deepen Students' Reasoning "(Doing and talking Math and Science: Strengthening Reasoning, Strengthening Language)

Grade 8 Sample

The information that follows provides an example of how the identified priority math standards that fall under the identified priority clusters can be addressed to support students gaining proficiency in English.

GRADE 8 MATH STANDARD 8.F.B.4

Priority Content for 8th Grade

Content Cluster: 8.F.B

Math Standard Addressed: 8.F.B.4
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Standards for Mathematical Practice

SMP 1 - Make sense of problems and persevere in solving them.

SMP 2 - Reason abstractly and quantitatively.

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP 6 - Attend to precision.

Intersection of English Language Development (ELD) and Mathematical Comprehension

Align ELD and Standards for Mathematical Practice

Standards for Mathematical Practice
Working collaboratively provides students the opportunity to use language while examining the problem presented (SMP 1), offering their opinions, and asking and answering questions (SMP 2) to develop their understanding of the mathematical concepts deeply as they solve cases (SMP 6). As students present their solutions, they adapt their language choices, as well as use specific nouns and noun phrases, adding details to accurately (SMP 6) and convincingly present their arguments (SMP 3). While listening attentively to their peers, they analyze and evaluate how well their peers present and support their arguments (SMP 3).

ELD Standards

PI.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics.

PI.3 - Offering and justifying opinions, negotiating with and persuading others in communicative exchanges.

PI.4 - Adapting language choices to various contexts (based on task, purpose, audience, and text type).

IP.5 - Listening actively to spoken English in a range of social and academic contexts.

PI.6 - Reading closely literary and informational texts and viewing multimedia to determine how meaning is conveyed explicitly and implicitly through language.

PI.7 - Evaluating how well writers and speakers use language to present or support ideas.

PI.8 - Analyzing how writers and speakers use vocabulary and other language resources for specific purposes (to explain, persuade, entertain, etc.) depending on modality, text type, purpose, audience, topic, and content area.

P1.9 - Expressing information and ideas in formal oral presentations on academic topics.

PII.3 - Using verbs and verb phrases.

PII.4 - Using nouns and noun phrases.

PII.5 - Modifying to add details.

GRADE 8 MATH STANDARD 8.F.B.4 (continued)

Examples of Correct Student Thinking and Misconceptions

Sample Starter Problem
"Tonya is looking at a graph that shows a line drawn between two points with a slope of -5. One of the points is smudged and she cannot read it. The points as far as she can tell are (3,5) and (x, 10). What must the value of x be? Explain."

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Classwork should not be limited to memorizing facts, rules, or only carrying out calculations, but should extend to showing, describing, and discussing the underlying mathematical meaning of those procedures. Provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, tables, graphs, words, etc.).
2. Support productive struggle in learning mathematics. For example, give students time to struggle with tasks, and ask questions that scaffold students' thinking without stepping in to do the work for them.
   - To develop SMP 1, ask "What information is given in the problem?"
   - To develop SMP 2, ask "What is the relationship of the quantities?"
   - To develop SMP 3, ask "What mathematical evidence would support your solution?"
   - To develop SMP 6, ask "How are you showing the meaning of the quantities?"
3. Create an anchor chart with the students highlighting that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Overarching Instructional Guidelines

Adapted from enVision Mathematics
Grade 8, Topic 3, Lesson 4

Scaffolding

- Focus II: Scaffolding and Supports for Simultaneous Development
- Guideline 4: Opportunities for students to interact with and produce a variety of methods and representations.
- Specification 4A: Learning activities provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, tables, graphs, words, etc.).

Principles to Actions: Ensuring Mathematical Success for All (NCTM)

- Mathematics Teaching Practices: Support productive struggle in learning mathematics.

Leading with Learning (WestEd)

- Guiding Questions to Develop Mathematical Practices

GRADE 8 MATH STANDARD 8.F.B.4 (continued)

Examples of Correct Student Thinking and Misconceptions

Example of Correct Student Thinking

10 - 5 = -5,
X - 3

-5(x - 3) = 5,
-5x + 15 = 5,
-5x = -10,

x = 2

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Nurture opportunities for teacher-student and student-student interactions that model and reflect the intent of the Mathematical Practices. Have students work in small groups to analyze the graph when describing its functional relationship between two quantities. Ask them to indicate where the function is increasing or decreasing, and provide justification as to whether it is a linear or nonlinear function.
2. Group Activity: Problem-Solving Quadrant is a tool for structuring students' engagement with mathematics by creating and connecting multiple representations of a mathematical relationship.

Students work in groups of four to solve a math problem using multiple representations based on "Rule of Four." Each group member is assigned a specific role: Graphical, Numeric, Symbolic, and Words. Students may either choose their roles or the teacher can assign roles considering students' strengths and areas of growth. Over time, the teacher should be vigilant to make sure that ELs are encouraged and supported to do more language-intensive roles (e.g., Words), keeping in mind that all roles require language to communicate mathematical thinking. The independent part of this activity allows students to practice creating a specific representation from among the Rule of Four. Then, the pair/group sharing part of the activity allows students to see and hear about other representations provided by their group members. Students can use Problem-Solving Quadrant Work Space to create a response and take notes during the sharing part of this activity. The activity allows time, space, and structure for students to engage in discourse by sharing their representations, justifying their responses, and making connections between different representations.

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

Nurture opportunities

- Focus V: Assessment of Mathematical Content, Practices, and Language
- Guideline 13: Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency
- Specification 13A: Provide examples of teacher-student and student-student interactions that model and reflect the intent of the Mathematical Practices.

Group Activity: Problem-Solving Quadrant

- Focus II: Scaffolding and Supports for Simultaneous Development
- Guideline 4: Opportunities for students to interact with and produce a variety of methods and representations.
- Specification 4A: Learning activities that provide ways for students to generate and interpret a range of mathematical methods and representations (symbols, manipulatives, graphs, tables, words, etc.).
- Specification 4B: Teacher materials provide guidance to encourage students to draw comparisons and connections across different methods and representations.

GRADE 8 MATH STANDARD 8.F.B.4 (continued)

Examples of Correct Student Thinking and Misconceptions

Non-example (Misconception) of Student Thinking

A student finds the value of x to be -22.

```
x - 3 = -5
10 - 5

x - 3 = -5
5

x - 3 = 5 x -5
x - 3 = -25
x = -22
```

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

1. Create opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.
2. Have students justify their reasoning, communicate their reasoning to others, and respond to the arguments of others. This includes explaining the reasoning behind correct answers, as well as the misconceptions behind incorrect responses, which enhances conceptual understanding of central math ideas.
3. During lessons, have students contrast correct and incorrect ways to solve a problem. Ask them to talk explicitly about why a particular misconception occurs, how to avoid it, and how to think correctly about the mathematics in the problem.
4. Check that the student has used the correct process. Ask them to explain how they calculated the slope of a line given two points on the line. They should respond:

   ```
   Difference in the y-values
   Difference in the x-values
   ```

5. In addition, ask the student to tell you the equation that they would use to find the missing x-value.

   ```
   10 - 5 = -5,
   X - 3
   ```

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (ELSF)

Mathematical errors

- Focus III: Mathematical Rigor through Language
- Guideline 7: Explicit guidance for teachers to engage students in using mathematical practices.
- Specification 7B: Teacher materials point out opportunities for students to evaluate and address mathematical errors, misconceptions, and clarity of communication.

A Framework for Re-envisioning Mathematics Instruction for English Language Learners (CGCS)

- Encouraging Productive Struggle

GRADE 8 MATH STANDARD 8.F.B.4 (continued)

Additional Scaffolding and Supports for Simultaneous Development

Vocabulary Development
Front-load key vocabulary that students will need to understand and use in the lesson. Put the words in context in student-friendly language along with examples or visuals when possible.

Provide opportunities to draw on and incorporate students' backgrounds and lived experiences into their mathematics learning.
Discuss slope lines related to community demographics, immigration, economic growth or decline, health-related issues, etc.

Overarching Instructional Guidelines

General Template Instructions

The steps below can be used by teachers in the creation of their own tables.

GRADE _____ MATH STANDARD _____

Priority Content for _____ Grade

Content Cluster: _____

1. Identify the priority clusters for your grade level using the Priority Content doc.
2. Consider the scope and sequence for the year based on the priority clusters listed and how they relate to remaining grade-level content.
3. Identify which cluster you will address first based on assessed student needs.
4. Identify the specific CA Math Standards within the identified cluster..
5. Identify scope and sequence for the identified standards.
6. Identify the standard you will address.

Standards for Mathematical Practice

SMP 1 - Make sense of problems and persevere in solving them.

SMP 2 - Reason abstractly and quantitatively.

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP 4 - Model with mathematics.

SMP 5 - Use appropriate tools strategically.

SMP 6 - Attend to precision.

SMP 7 - Look for and make use of structure.

SMP 8 - Look for and express regularity in repeated reasoning.

Intersection of English Language Development (ELD) and Mathematical Comprehension

Align ELD and Standards for Mathematical Practice

7. Refer to Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning, for support in aligning ELD standards and Standards for Mathematical Practice to the identified priority content standard.
8. Identify priority ELD standards and language objectives that are detailed in selected standards.

ELD Standards

PI.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics.

PI.3 - Offering and justifying opinions, negotiating with and persuading others in communicative exchanges.

PI.4 - Adapting language choices to various contexts (based on task, purpose, audience, and text type).

IP.5 - Listening actively to spoken English in a range of social and academic contexts.

PI.6 - Reading closely literary and informational texts and viewing multimedia to determine how meaning is conveyed explicitly and implicitly through language.

PI.7 - Evaluating how well writers and speakers use language to present or support ideas.

PI.8 - Analyzing how writers and speakers use vocabulary and other language resources for specific purposes (to explain, persuade, entertain, etc.) depending on modality, text type, purpose, audience, topic, and content area.

P1.9 - Expressing information and ideas in formal oral presentations on academic topics.

PII.3 - Using verbs and verb phrases.

PII.4 - Using nouns and noun phrases.

PII.5 - Modifying to add details.

GRADE _____ MATH STANDARD _____ (continued)

Examples of Correct Student Thinking and Misconceptions

Sample Starter Problem

9. Refer to your adopted math curriculum to identify common student thinking that can provide an example and non-example for the identified standard.
10. Also consider using CAASPP Practice Test or Interim Assessment items as samples.
    a. Many publishers offer a variety of story situations as the context for the problems presented to students. This can prove confusing early on in students' conceptual understanding.
    b. Students benefit from analyzing the same context before moving on to more abstract thinking.
    c. For example, in the punch sample problem in the 6th grade, try changing the numerical properties before switching from juice and soda.
    d. Then students aren't having to switch their focus to understand the context, and can focus on mathematical concepts.

Example of Correct Student Thinking

11. Select several models of correct student thinking. Present them separately so students can analyze them individually.

Non-example (Misconception) of Student Thinking

12. Select a non-examples to support students with error analysis, as well as:
    a. constructing viable arguments and critiquing the reasoning of others.
    b. attending to precision.

Actionables: Why & How Scaffolding and Supports for Simultaneous Development

Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.

14. Select resources to support error analysis of identified priority standards. Ask yourself, how do these scaffolds and activities support students with ELD and math priority standards?

    Appropriate, integrated ELD support that targets students' proficiency level is required for ELs. Scaffolding activities can be used to support other learners.

    Consider remote learning application of activities and supports:

    Do's & Dont's: Remote Math Instruction for ELs

    Considerations when Adapting to Online Learning for English Learners

Overarching Instructional Guidelines

Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

13. Review resources to strategically select scaffolds and activities to support students with priority content standards.

    Attaching activities to instructional guidelines can support focus when selecting purposeful activities.

    One way to do this is by using the EL Success Forum's Math Guidelines Resources
    a. Access Math Guidelines PDF (p. 9) to select areas of focus and guidelines (1–15).
    b. Go to the Explore the Resources page, and select math, grade level, and guideline to view resources.
    c. Select strategic resources to support error analysis of identified priority standard (enter into #14)

Other sources:

National Council of Teachers of Mathematics, Principles to Actions Professional Learning Toolkit

WestEd, Leading with Learning[1] (Educator Resources)

WestEd, Math Pathways & Pitfalls[2] (Sample Lessons)

[1] "Leading with Learning," Copyright 2018 by WestEd. Reprinted by permission of WestEd.
[2] "Math Pathways & Pitfalls," Copyright 2008 by WestEd. Reprinted by permission of WestEd.

GRADE _____ MATH STANDARD _____ (continued)

Additional Scaffolding and Supports for Simultaneous Development	Overarching Instructional Guidelines
15. Are there any supports you want to include that do not fall under error analysis structure?	

General Template

GRADE _____ MATH STANDARD _____

Priority Content for _____ Grade

Content Cluster: _____

Standards for Mathematical Practice

SMP 1 - Make sense of problems and persevere in solving them.

SMP 2 - Reason abstractly and quantitatively.

SMP 3 - Construct viable arguments and critique the reasoning of others.

SMP 4 - Model with mathematics.

SMP 5 - Use appropriate tools strategically.

SMP 6 - Attend to precision.

SMP 7 - Look for and make use of structure.

SMP 8 - Look for and express regularity in repeated reasoning.

Intersection of English Language Development (ELD) and Mathematical Comprehension

Align ELD and Standards for Mathematical Practice

ELD Standards

PI.1 - Exchanging information and ideas with others through oral collaborative discussions on a range of social and academic topics.

PI.3 - Offering and justifying opinions, negotiating with and persuading others in communicative exchanges.

PI.4 - Adapting language choices to various contexts (based on task, purpose, audience, and text type).

IP.5 - Listening actively to spoken English in a range of social and academic contexts.

PI.6 - Reading closely literary and informational texts and viewing multimedia to determine how meaning is conveyed explicitly and implicitly through language.

PI.7 - Evaluating how well writers and speakers use language to present or support ideas.

PI.8 - Analyzing how writers and speakers use vocabulary and other language resources for specific purposes (to explain, persuade, entertain, etc.) depending on modality, text type, purpose, audience, topic, and content area.

P1.9 - Expressing information and ideas in formal oral presentations on academic topics.

PII.3 - Using verbs and verb phrases.

PII.4 - Using nouns and noun phrases.

PII.5 - Modifying to add details.

GRADE _____ MATH STANDARD _____ (continued)

Examples of Correct Student Thinking and Misconceptions	Actionables: Why & How Scaffolding and Supports for Simultaneous Development	Overarching Instructional Guidelines
Sample Starter Problem	**Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.**	Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

GRADE _____ MATH STANDARD _____ (continued)

Examples of Correct Student Thinking and Misconceptions	Actionables: Why & How Scaffolding and Supports for Simultaneous Development	Overarching Instructional Guidelines
Example of Correct Student Thinking	**Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.**	Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

GRADE _____ MATH STANDARD _____ (continued)

Examples of Correct Student Thinking and Misconceptions	Actionables: Why & How Scaffolding and Supports for Simultaneous Development	Overarching Instructional Guidelines
Non-example (Misconception) of Student Thinking	**Incorporate these practices into your instruction to make the math content more equitable and accessible for your ELs.**	Guidelines for Improving Math Materials for English Learners (English Learners Success Forum)

GRADE _____ MATH STANDARD _____ (continued)

Additional Scaffolding and Supports for Simultaneous Development	Overarching Instructional Guidelines

Resources

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Appendix

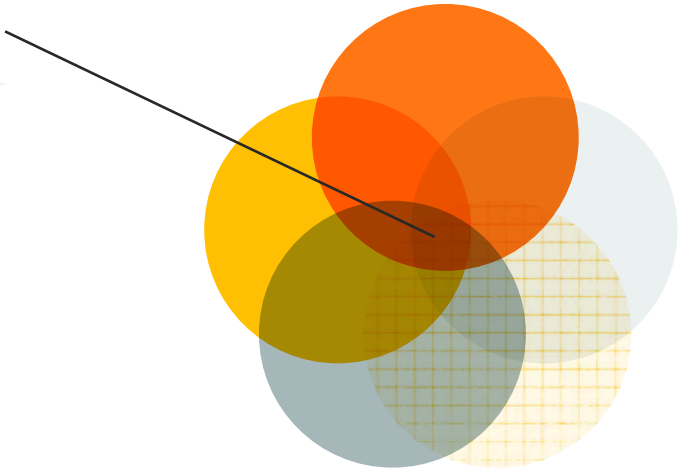
Summary of Resources Used by Tool Developers

This tool presents support, ideas, recommendations, and considerations for educators on how to scaffold language to develop mathematical concepts and communication, providing examples of effective classroom practices that align with key principles to effectively support English learners. One table for grades 6–8 is provided to serve as an example of principles and activities that classroom teachers can strategically use to support English learners in mathematics.

This document seeks to synthesize research surrounding effective instructional principles and practices for English learners, and apply them to Common Core Mathematics Priority Standards and CA ELD Standards in order to support teachers with developing students' understanding of mathematical concepts and with communicating their understandings.

Considerations for Supporting Priority Math Content for Students Gaining Proficiency in English in Grades 6-8

- Framework for Re-envisioning Mathematics Instruction for English Language Learners
- Guidelines for Improving Math Materials for English Learners
- English Learner Road Map
- 2020-2021 Priority Instructional Content in ELA/Literacy and Mathematics
- Integrating the CA ELD Standards into K-12 Mathematics and Science Teaching and Learning



The descriptions below summarize the sources incorporated in the creation of the tool.

The Council of the Great City Schools
"2016 Framework for Re-envisioning Mathematics Instruction for English Language Learners"
The "Key Instructional Principles and Practices" found in the 2016 "Framework for Re-envisioning Mathematics Instruction for English Learners" highlight the need for "understanding the interdependence of language and math in an era when the new college- and career-readiness standards in mathematics include unprecedented language demands."

A focus was placed on the practice of encouraging productive struggle. The Framework details that students must engage in rigorous grade-level tasks and assignments in which they are growing their conceptual understanding through productive struggle. The struggle is productive because students are supported using strategic scaffolds designed to grow their conceptual understanding. A result of this growth includes the application of academic language to explain, conjecture, and justify their reasoning during constructive conversations. The research surrounding effective instructional principles and practices for English learners was instrumental in creating the grade-level examples below.

English Learner Success Forum
"Guidelines for Improving Math Materials for English Learners"

The abovementioned "Key Principles" were echoed in the English Learner Success Forum's Guidelines for Improving Math Materials for English Learners "Areas of Focus" needed to provide quality supports for ELs:

- Mathematical Rigor through Language
- Interdependence of Mathematical Content, Practices, and Language
- Leveraging Students' Assets
- Assessment of Mathematical Content, Practices, and Language
- Scaffolding and Supports for Simultaneous Development

Additionally, the Guidelines document also provides examples and resources for teachers that address each of their Areas of Focus.

English Learner Road Map

California Department of Education's English Learner Roadmap is intended to provide guidance to local educational agencies on welcoming, understanding, and educating the diverse population of students who are English learners attending California public schools. Principle Two: Intellectual Quality of Instruction and Meaningful Access" was a guiding document for creating the tool.

Student Achievement Partners
"2020-2021 Priority Instructional Content in ELA/Literacy and Mathematics"

The mathematical priority clusters detailed in this document were used to select sample standards for the grades 6–8 examples below. This was done to support teachers with instructional decision making during the time of pandemic teaching, as an attempt to support the targeting of high-leverage standards without compromising rigor.

WestEd
"Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning"

"Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning" aligns math standards to ELD standards and includes mathematical practices and activity suggestions. The *Instructional Priority Content* document was combined and enhanced with guidance found in the Integrating document to support how priority standards can be addressed with ELD supports.

Alignment of Framework for Re-envisioning Mathematics Instruction for English Language Learners and Guidelines for Improving Math Materials for English Learners

Developers searched for alignment among the Framework's key principles and practices—designed to address the language demands in the new standards for mathematics that may pose challenges for students who are developing both English proficiency and academic language in mathematics—and ELSF's Guidelines for Improving Math Materials for English Learners in order to better understand how to incorporate the guidelines and key principles into the designing of this document.

ALIGNMENT

Framework for Re-envisioning Mathematics Instruction for English Language Learners	Guidelines for Improving Math Materials for English Learners
Employing Rigorous Tasks and Assignments Tasks and assignments for ELs and all math learners should be at a high level of cognitive demand, mathematically rigorous, on grade level, and make explicit connections between new and prior concepts. This may require multiple entry points, along with other appropriate supports for language development and communicating their understanding of mathematics, to allow for productive struggle while maintaining the high cognitive demand of the task.	**Area of Focus III: Mathematical Rigor through Language** 7. Explicit guidance for teachers to engage students in using mathematical practices. 8. Maintain appropriate challenge and high expectations of mathematics learning for EL students. 9. Guidance for facilitating mathematical discussion and co-construction of meaning.
Encouraging Productive Struggle Allowing students sufficient time to make sense of a task or problem before intervening. Teachers should resist the urge to lighten productive struggle, and instead, look for ways to retain the productive nature of the struggle.	**Area of Focus III: Mathematical Rigor through Language** 7. Explicit guidance for teachers to engage students in using mathematical practices. 8. Maintain appropriate challenge and high expectations of mathematics learning for EL students. 9. Guidance for facilitating mathematical discussion and co-construction of meaning. **Area of Focus IV: Leveraging Students' Assets** 10. Opportunities to draw on and incorporate students' cultural backgrounds and lived experiences in mathematics learning. 11. Suggestions for incorporating and valuing ELs' written and spoken contributions. 12. Encouragement for ELs to use and build on existing language resources.

ALIGNMENT (continued)

Framework for Re-envisioning Mathematics Instruction for English Language Learners

Employing Multiple Modes and Representations in Mathematics

- Universal Design for Learning (UDL) opportunities for engagement, representation, action, and expression to help advance students' understanding of mathematics.
- Teachers employ multiple modes of written and oral communication, and multiple representations.
- Teachers provide varied opportunities to participate in the classroom using concrete tools, pictorial representations, computers, assistive and instructional technologies, and manipulatives.
- Students are actively engaged in learning and develop the confidence to communicate their mathematical understanding in different modes and representations, using both informal and more formal language.
- Once the door of access and understanding is open, ELs can further develop academic language and use it to engage in mathematical discourse.

Guidelines for Improving Math Materials for English Learners

Area of Focus I: Interdependence of Mathematical Content, Practices, and Language

1. Strategic opportunities to use and refine both language and mathematics over time.
2. Explicit mathematics and language learning goals and pathways.
3. Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms.

Area of Focus II: Scaffolding and Supports for Simultaneous Development

4. Opportunities for students to interact with and produce a variety of methods and representations.
5. Directions for providing specialized individual and small group instruction to ELs.
6. Guidance for anticipating potential language demands and opportunities in student activities.

Area of Focus IV: Leveraging Students' Assets

10. Opportunities to draw on and incorporate students' cultural backgrounds and lived experiences in mathematics learning.
11. Suggestions for incorporating and valuing ELs' written and spoken contributions.
12. Encouragement for ELs to use and build on existing language resources.

Area of Focus V: Assessment of Mathematical Content, Practices, and Language

13. Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency.
14. Assessments able to capture and measure students' mathematics and language progress over time.
15. Guidance for recognizing and attending to student language produced to inform instructional decisions.

ALIGNMENT (continued)

Framework for Re-envisioning Mathematics Instruction for English Language Learners

Supporting Academic Language and Conversations

- Academic discussions focused on mathematical ideas also support exploratory and explanatory talk and writing.
- Precise and informal math language are important.
- In "academic mathematical discussions," we refer not only to students sharing their solutions to a problem, but discussions where students are supported by the teacher in gradually developing more sophisticated language to articulate their mathematical reasoning, and in deepening the understanding of other students through purposeful teacher or peer questions, focused on the mathematics and the mathematical reasoning.
- Serve as formative assessment to guide supports.
- Meaningfully interact by explaining, clarifying, justifying, and adding to the thinking of others.

Guidelines for Improving Math Materials for English Learners

Area of Focus I: Interdependence of Mathematical Content, Practices, and Language

1. Strategic opportunities to use and refine both language and mathematics over time.
2. Explicit mathematics and language learning goals and pathways.
3. Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms.

Examples and Resources:

- *Vocabulary Pieces, Roots, And Families; Mathematically Speaking; Strategic Grouping for Home Language Supports (Chval, Pinnow & Thomas, 2014; Vomvoridi-Ivanović & Chval, 2014)*
- *Spiralling Math and Language Content; Analyzing Content and Language Demands*
- *Anchor Charts: A Vocabulary Strategy; Bounce Cards for Primary Grades (Spanish), Primary Grades (English), Intermediate Grades (English), Spiralling Math and Language Content; Talk Moves*
- *Are my ELs Attaching Meaning to Math Words?*

Area of Focus III: Mathematical Rigor through Language

7. Explicit guidance for teachers to engage students in using mathematical practices.
8. Maintain appropriate challenge and high expectations of mathematics learning for EL students.
9. Guidance for facilitating mathematical discussion and co-construction of meaning.

Area of Focus IV: Leveraging Students' Assets

10. Opportunities to draw on and incorporate students' cultural backgrounds and lived experiences in mathematics learning.
11. Suggestions for incorporating and valuing ELs' written and spoken contributions.
12. Encouragement for ELs to use and build on existing language resources.

Area of Focus V: Assessment of Mathematical Content, Practices, and Language

13. Descriptions, illustrations, and examples of quality work and mathematical practices with varying levels of language proficiency.
14. Assessments able to capture and measure students' mathematics and language progress over time.
15. Guidance for recognizing and attending to student language produced to inform instructional decisions.

ALIGNMENT (continued)

Framework for Re-envisioning Mathematics Instruction for English Language Learners

Using Strategic Scaffolding

- When necessary, teachers should strategically employ scaffolds specifically targeted to meet an individual student's educational needs or academic difficulties, while ensuring that this scaffolding does not compromise their access to rigorous mathematics content or their development of higher-order conceptual understanding.
- The concept of scaffolding is often misunderstood and misinterpreted.
- It provides an entry point for students to actively engage with cognitively demanding grade-level mathematics and empowers students to engage in, and ultimately emerge successfully from, productive struggle.
- It is specifically targeted to reflect an understanding of students' previous experiences with mathematics instruction, their language development history, and their educational needs.
- It should enable all students to be active participants in the mathematics classroom.

Guidelines for Improving Math Materials for English Learners

Area of Focus I: Interdependence of Mathematical Content, Practices, and Language

1. Strategic opportunities to use and refine both language and mathematics over time.
2. Explicit mathematics and language learning goals and pathways.
3. Regular and varying opportunities to learn, reflect upon, and demonstrate learning of mathematics using a variety of modes and forms.

Area of Focus II: Scaffolding and Supports for Simultaneous Development

4. Opportunities for students to interact with and produce a variety of methods and representations.
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Area of Focus IV: Leveraging Students' Assets

10. Opportunities to draw on and incorporate students' cultural backgrounds and lived experiences in mathematics learning.
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