A Pathway to Equitable Math Instruction

Fostering Deep Understanding

Methods for deepening student conceptual understanding through orchestrated math discussions that build on and connect multiple strategies.

STRIDE

2

Fostering Deep Understanding

The purpose of this tool is to highlight the diversity of student thinking, misconceptions, alternate solutions, and connections so any student, regardless of level, can contribute in meaningful discussion and gain agency and deep conceptual understanding.

Teachers also build pedagogical content knowledge, cultivate the flexibility to work with diverse students, and practice continuous improvement.

This tool is designed for middle school math teachers, but can be adapted for any grade level. It is used for planning the lesson, data collection during the lesson, as well as reflection after the lesson and for future planning. It is a cyclical, living practice of continuous improvement.

HOW TO USE THIS TOOL

To use this tool, follow the **Planning Checklist**, which breaks down the process into four distinct parts: Understand the Activity, Plan the Activity, Collect Student Samples and Thinking During the Lesson, and Reflect on Content Understanding After the Lesson.

First, **Understand the Activity** by reading 5 Practices for Orchestrating Productive Mathematics Discussions, how to choose cognitively demanding tasks, and how to facilitate equitable student discussions and honor student responses.

Next, **Plan the Activity** by identifying a specific learning goal, task, content, and math practice. Anticipate student strategies that may be used, practice sequencing student thinking, and consider virtual adaptations and necessary adjustments.

Then, **During the Lesson**, collect student samples and data, and guide student discussion.

And finally, **After the Lesson**, examine what student data says about your students' current level of understanding, identify student responses you did not anticipate or understand, and identify potential areas of development for your own content understanding.

THEMES

Content + Conceptual Understanding

GUIDING PRINCIPLES

Equitable access to grade-level priority math standards.

Learning opportunities for students to engage with the standards for mathematical practice.

Assets-based formative assessments to inform instruction.

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Equitable Math Discussions through Student Discourse

Introduction

Have you ever taught a lesson and then encountered students solving the presented problems in a different way?
Do opposing student thoughts scare you or excite you?
Is your first reaction to differing thoughts “Tell me more!” or “Let me show you”?

Purpose

The purpose of this tool is to highlight the diversity of students’ thinking, misconceptions, alternative solutions, and connections so any student, regardless of level, can contribute and gain equitable access to grade-level content. Students gain agency, voice, and deep content knowledge through carefully planned discussions. Teachers also build deep content knowledge, cultivate the flexibility to work with diverse students, and practice continuous improvement.

This tool is meant to be used with *all grade levels and content*. It is used for *planning* the lesson, data collection *during* the lesson, as well as reflection after the lesson and for future planning. It is a cyclical, living practice of continuous improvement. For a detailed demonstration on how to use this tool, see the short video here.

WHO	BENEFITS	HOW
STUDENT	Gains equitable access to meaningful and engaging grade-level content.	Represent *all* levels of student thought: initial thoughts, misconceptions, alternative solutions, connections.
	Gains agency and uses their voice.	Center content understanding around *student* questions, diverse thinking, misconceptions, and alternative strategies.
	Builds deep content knowledge.	Facilitate active sharing, discussion, and connections between different students' thinking.
TEACHER	Builds deep content knowledge.	Intentional anticipating and sequencing potential student thinking to reach learning goals.
	Gains flexibility when confronted with diverse or unexpected thinking.	Explore diverse student thinking through class discussion and questioning.
	Practices continuous improvement.	Reflect before, during, and after the lesson to grapple with your own development needed for the content.

Getting Started

Understand the *5 Practices for Orchestrating Productive Mathematics Discussions*

5 Practices for Orchestrating Productive Mathematics Discussions, by Margaret Schwan Smith and Mary Kay Stein[1], is the foundational reference for this tool. It involves five stages for highlighting and sequencing student thinking:

1. **Anticipate** likely student emerging ideas and alternative solutions to mathematical tasks.
2. **Monitor** students' actual responses to the tasks.
3. **Select** student responses to feature during the discussion.
4. **Sequence** student responses in a purposeful order to build a coherent math story.
5. **Connect** different students' responses through math discussion.

Understanding this practice is the major step to begin implementation. If you are not familiar with the 5 *Practices*, it is recommended that professional development, coaching, and independent research be coupled with this resource. Level 1 and 2 readings are **must reads**, and level 3 is an optional deep dive.

LEVEL 1 INTRODUCTION	5 Practices for Orchestrating Productive Mathematics Discussions (SFUSD)
LEVEL 2 IMPLEMENTATION	Chapter 1: Introducing the Five Practices
OPTIONAL LEVEL 3 DEEP DIVES	The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion

[1] Margaret Schwan Smith and Mary Kay Stein, *5 Practices for Orchestrating Productive Mathematics Discussions* (Reston, VA: National Council of Teachers of Mathematics, 2011).

Using the Planning Checklist

In order to use this tool, follow the **Planning Checklist**, which breaks down the process into four distinct parts: Understand the Activity, Plan the Activity, Collect Thinking During the Lesson, and Reflect After the Lesson.

Understand the Activity by reading 5 Practices for Orchestrating Productive Math discussions, how to choose cognitively demanding tasks, and how to facilitate equitable student discussions and honor student responses.

Then, **Plan the Activity** by identifying a specific learning goal, task, content, and math practice. Anticipate student strategies that may be used, practice sequencing student thinking, and consider virtual adaptations and necessary adjustments.

Then, **During the Lesson** collect student samples and data and guide student discussion. And finally, **After the Lesson**, examine what student data says about your students' current level of understanding, identify student responses you did not anticipate or understand, and identify potential areas of development for your own content understanding.

Planning Checklist

UNDERSTAND THE ACTIVITY (Before the Lesson)

- [ ] **1. Understand the 5 Practices for Orchestrating Productive Mathematics Discussions.**
  "Chapter 1: Introducing the Five Practices"

- [ ] **2. Understand how to choose cognitively demanding tasks.**
  A. What are cognitively demanding tasks?
  B. Libraries for finding tasks: Math Assessment Project or Illustrative Mathematics

- [ ] **3. Understand how to facilitate equitable student discussion and honor student responses.**
  A. Facilitating Effective Discussions
  B. Equitable Discussion in a Multicultural Classroom

- [ ] **4. Choose tasks.**
  Consider Priority Standards by Grade Level.
  Consider 2020-21 Priority Instructional Content in Mathematics.

PLAN THE ACTIVITY (Before the Lesson) USE TEMPLATE BELOW

- [ ] **1. Identify a specific learning goal, task, content standards, and mathematical practice.**

- [ ] **2. Anticipate student strategies that may be used to complete the task.**

- [ ] **3. Practice sequencing student thinking.**
  A. Complete the task with various colleagues and sequence solutions.
  B. Review Chapter 1 or the 5 Practices

- [ ] **4. Consider virtual adaptations or other necessary adjustments.**
  (See "Resources" at the end of the Toolkit.)

COLLECT STUDENT SAMPLES AND THINKING (During the Lesson)

- [ ] **1. Collect student samples and data.**

REFLECT ON STUDENT AND TEACHER CONTENT UNDERSTANDING (After the Lesson)

- [ ] **1. Examine what student data says about your students' current level of understanding.**

- [ ] **2. Identify student responses you did not anticipate or understand.**

- [ ] **3. Identify potential areas of development for your content understanding.**

Equitable Math Discussions Activity Template

For a detailed demonstration on how to use this tool, see the short video here.

1. CONTENT PREPARATION (Use Priority Standards by Grade Level.)

LEARNING GOAL	
LEARNING TASK	
CONTENT STANDARDS	
8 STANDARDS FOR MATH PRACTICE	☐ 1. Make sense of problems and persevere in solving them. ☐ 2. Reason abstractly and quantitatively. ☐ 3. Construct viable arguments and critique the reasoning of others. ☐ 4. Model with mathematics. ☐ 5. Use appropriate tools strategically. ☐ 6. Attend to precision. ☐ 7. Look for and make use of structure. ☐ 8. Look for regularity in repeated reasoning.

2. ANTICIPATED STUDENT STRATEGIES SEQUENCING (Use Progression Documents by Topic.)

ORDER Sequence strategies for learning target	ANTICIPATED STRATEGY Anticipate the various strategies/methods your students will apply to arrive at a solution. One strategy per row.	WHO / OBSERVATIONS Which students are using the strategy?

3. REFLECT UPON YOUR LESSON AND PLAN NEXT STEPS

TASK	REFLECTION NOTES	NEXT STEPS
Examine what student data says about your students' current level of understanding.		
Identify student responses you did not anticipate or understand.		
Identify potential areas of development for your content understanding.		

Resources

1. Understand the Activity

5 Practices for Orchestrating Productive Mathematics Discussions

a. Level 1: 5 Practices for Productive Math Discussions Overview (SFUSD)
b. Level 2: Using the 5 Practices in Mathematics Teaching (NCTM 2018)[2]
c. Level 3: *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein Book)

2. Understand the Content

Priority Standards by Grade Level

a. Achieve the Core Priority Standards by Grade Level Website
b. Achieve the Core Priority Standards by Grade Level 2020-2021 PDF

Content Standards

Content Standards CCSS
Standards for Mathematical Practice CCSS

3. Understand the Sequence

Progression Documents by Topic

a. Achieve the Core Progressions by Topic
b. UnboundEd Content Guides

4. Cognitively Demanding Tasks

a. What are cognitively demanding tasks?
b. Math Assessment Project
c. Illustrative Mathematics

5. Facilitate Small Group Discussion

a. Complex Instruction
b. Facilitating Effective Discussions
c. Equitable Discussion in a Multicultural Classroom

6. Virtual Adaptations

a. **Select and sequence asynchronously.**
Give students tasks asynchronously and submit to the teacher. The teacher selects, sequences, and prepares an asynchronous or synchronous lesson around students' responses.

b. **Select and sequence using Google Slides deck.**
Provide a shared Google Slides deck for students to populate with their thinking by typing on their assigned slides or uploading a picture of their work. The teacher can then select and sequence the slides, and share with students synchronously or asynchronously.

c. **Select and sequence with interactive whiteboards.**
Use interactive whiteboards to select, sequence, and share solutions.

[2] Keith Nabb, Erick B. Hofacker, Kathryn T. Ernie, and Susan Ahrendt, "Using the 5 Practices in Mathematics Teaching," *Mathematics Teacher* Vol. 111, No. 5 (March 2018): 366-373.

EXAMPLE

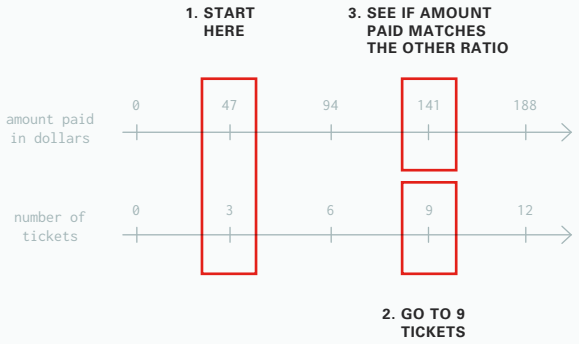
Equitable Math Discussions Activity Template

For a detailed demonstration on how to use this tool, see the short video here.

1. CONTENT PREPARATION (Use Priority Standards by Grade Level.)

LEARNING GOAL	I can decide whether or not two situations are happening at the same rate.
LEARNING TASK	Diego paid $47 for 3 tickets to a concert. Andre paid $141 for 9 tickets to a concert. Did they pay at the same rate? Explain your reasoning. Students will work individually. Their solutions will be shared anonymously. Students will compare their strategies at the end of the lesson.
CONTENT STANDARDS	**6.RP.A.2 Ratios and Proportional Relationships** Understand the concept of a unit rate a/b associated with a ratio a:b with b0, and use rate language in the context of a ratio relationship. Expectations for unit rates in this grade are limited to non-complex fractions.
8 STANDARDS FOR MATH PRACTICE	1. Make sense of problems and persevere in solving them. ● 2. Reason abstractly and quantitatively. ● 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for regularity in repeated reasoning.

2. ANTICIPATED STUDENT STRATEGIES SEQUENCING (Use Progression Documents by Topic.)

ORDER Sequence strategies for learning target	ANTICIPATED STRATEGY Anticipate the various strategies/methods your students will apply to arrive at a solution. One strategy per row.	WHO / OBSERVATIONS Which students are using the strategy?
1		Student A Student C Student J Student L Student P Student Q
2	Since 9 is 3 x 3, multiply 47 by 3, 47 x 3 = 141 . Diego would have paid $141 for 9 tickets if he paid at the same rate he did for 3 tickets. Since this is what Andre paid for 9 tickets, they paid at the same rate.	Student B Student D Student K Student M Student N
3	47 ÷ 3 = $15.67 , 141 ÷ 9 = $15.67 Each ticket costs $15.67, they paid the same rate	Student E Student F Student H Student Q

3. REFLECT UPON YOUR LESSON AND PLAN NEXT STEPS

TASK	REFLECTION NOTES	NEXT STEPS
Examine what student data says about your students' current level of understanding.	*The students are in different places along the continuum. There were 4 students who leaned on additive reasoning rather than multiplicative reasoning. I was able to connect their reasoning to the double number line and model the multiplicative reasoning.* *Once we discussed the double number line, I connected it to the method of multiplying by a factor of 3 and compared it to the division model.* *We concluded with the unit rate strategy.* *This student data tells me that students are in various places. I intentionally started with the misconception, (i.e. using additive reasoning) and moved from representational to abstract, so that students can make connections between their method and the more abstract methods.* *The goal for this unit is to build conceptual understanding, so I will continue to make connections between the various approaches.*	The next lesson introduces ratio tables. I need to be sure that I show the relationship between each of the methods shown today and ratio tables. I also can help the students who relied on additive reasoning to see how they can translate to multiplicative reasoning using a ratio table.
Identify student responses you did not anticipate or understand.	**Misconception:** *Using additive reasoning* 3 + 6 = 9 *and* 47 + 94 = 141 *(got stuck)* (4 students: Students G, I, R, S)	I will make the connection for students who relied on additive reasoning to see how they can translate to multiplicative reasoning using a ratio table.
	47 goes into 141, 3 times and 3 goes into 9, 3 times, therefore they paid the same price. (3 students: Students T, U, V)	I will show the students who divided that they are using multiplicative reasoning.
Identify potential areas of development for your content understanding.	*I need to push myself to continue to approach ratios in a conceptual way, rather than rushing to teach a procedure. I want to continue to understand the connections between the various strategies used to solve these types of problems. I need to study how ratios will show up in 7th grade, and make sure that I am setting students up for the 7th-grade approach.* **Discussion for Professional Learning Community (PLC):** • How can I "honor" a wrong strategy? • Is there an equity-oriented way to approach wrong strategies? • When does the additive strategy apply or make sense? • Can I think of a time or example when that strategy does work or apply or make sense? • Why do we as math teachers see this an absolute correct approach?	